

Rate equations of vertical-cavity semiconductor optical amplifiers

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We rigorously establish the rate equations for vertical-cavity semiconductor optical amplifiers, starting from a general energy rate equation. Our results show that the conventional rate equation used so far in the literature is incorrect because of an inappropriate calculation of the mirror losses. Our calculations include the effect of amplified spontaneous emission and can be used to describe the properties of resonant-cavity-enhanced photodetectors. © 2002 American Institute of Physics. [DOI: 10.1063/1.1476056]

Vertical-cavity semiconductor optical amplifiers (VCSOAs) have emerged recently as one important family of optoelectronic devices. Indeed, conventional in-plane semiconductor optical amplifiers (SOAs), unless specially designed, show polarization sensitivity and high coupling losses to optical fibers. Because of their vertical and circular cavity geometry, VCSOAs can inherently overcome these problems while offering the possibility of parallel processing and testing. They, moreover, have the advantage of a low-noise figure compared to SOAs.¹

The models proposed for the design of in-plane SOAs (Refs. 2–4) cannot be readily extended to VCSOAs. Indeed, because of their small gain per pass, VCSOAs require mirrors with high reflectivity, which is usually obtained by taking advantage of additive interferences in periodic dielectric structures, known as dielectric Bragg reflectors (DBRs). The amplitude and phase of these mirrors vary strongly with the angle and wavelength. In addition, because of the strong feedback provided by the mirrors, a standing optical wave is created in the cavity. This requires careful positioning of the active layers inside the cavity so as to benefit from the gain-enhancement mechanism.⁵ Finally, because the device has to be biased above material transparency, amplified spontaneous emission (ASE) is emitted and cannot be neglected close to the laser threshold. The standard way of calculating the ASE in in-plane SOAs is to use a traveling rate equation for the field intensity.^{4,6} This is justified as long as the active region is longer than the wavelength of the cavity modes. For VCSOAs, since the active region is very thin, it is important to consider the field rather than the powers in order to correctly take into account interference effects.

Performance predictions for VCSOAs were first proposed by Tombling,¹ based on the work of Mukai and Yamamoto³ using semiconductor rate equations for calculating the gain of the structure. Karlsson and Höijer⁷ investigated the detection and amplification characteristics of VCSOAs using the Fabry–Pérot approach initially derived by Adams⁶ for Fabry–Pérot laser amplifiers (FPLAs). Recently, Piprek, Björlin, and Bowers proposed a detailed analysis of VCSOAs, based on the rate equation approach.⁸ The rate equation and the Fabry–Pérot methods are known to

give significantly different predictions, especially in the gain saturation regime.^{2,8} This mechanism occurs as high-signal power is injected in the amplifier or close to the laser threshold: because of the corresponding increase of stimulated emission, the carrier density in the active region is reduced. This decreases the material gain and then the gain of the amplifier.

It is then of great interest to understand the origin of this discrepancy and to rigorously establish the rate equations for VCSOAs, which is the main objective of this letter. Our results show that the conventional rate equation used so far in the literature is incorrect for below threshold analysis of the mirror losses. Our calculations include the effect of amplified spontaneous emission. We conclude this letter demonstrating that our results can be readily applied to resonant-cavity-enhanced photodetectors (RCEPDs), which can be considered as VCSOAs with negative material gain.

For VCSOAs, the active region is usually made of thin quantum wells placed at the antinode of the electric field in the cavity. Provided the injection is uniform in the cavity and lateral effects are neglected (such as lateral carrier diffusion), the material gain can then be assumed to be independent of the position. This simplifies considerably the problem of calculating both the amplified spontaneous emission and the gain of the amplifier. This approximation allows us to use a single energy rate equation for the photons,^{5,9} which is expressed as

$$\frac{\partial W_{\text{tot}}}{\partial t} = G_{\text{tot}} - P_{\text{tot}}, \quad (1)$$

where $W_{\text{tot}} = h\nu V_p N_p$ is the total energy stored in the structure, $h\nu$ the energy of the mode, V_p the volume occupied by the photons, and N_p the photon density in the active region. The total generation rate can be expressed as the sum of three contributions: $G_{\text{tot}} = G_{\text{tot}}^+ + G_{\text{tot}}^- + G_{\text{sp}}$, where G_{tot}^+ and G_{tot}^- are the generation and absorption rates related to the gain and internal losses in the structure, and G_{sp} the power generated inside the structure by the spontaneous emission and directed into the optical mode. With these definitions, the modal gain can be expressed^{5,9} as $\langle g \rangle = G_{\text{tot}}^+ / (v_g W_{\text{tot}})$ and the internal losses as $\alpha_i = -G_{\text{tot}}^- / (v_g W_{\text{tot}})$. The confinement factor Γ is by definition the ratio between the volume V occupied by the carriers in the active region, and the volume V_p oc-

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cupied by the photons. It is related to the modal gain according to $\Gamma = \langle g \rangle / g = V/V_p$. The total power emitted by the device is given by $P_{\text{tot}} = P_R + P_T + P_I + P_{\text{sp}}$, where P_R , P_T , and P_I are the powers reflected, transmitted, and injected in the device, respectively, and P_{sp} the spontaneous emission power radiated outside the cavity and in the optical mode.

Using the above definitions and energy rate Eq. (1), one finds the general rate equation for the photons⁵ $\partial N_p / \partial t = (\Gamma g - \alpha_i - \alpha_m) v_g N_p + \Gamma \beta_{\text{sp}} R_{\text{sp}}$ where α_m is the mirror losses defined by $\alpha_m = (P_R + P_T - P_I + P_{\text{sp}}) / (v_g h \nu V_p N_p)$. The spontaneous emission factor β_{sp} describes the fraction of total spontaneous emission that radiates into the considered optical mode. The total spontaneous emission rate R_{sp} is related to the spontaneous generation rate according to $G_{\text{sp}} = h \nu V \beta_{\text{sp}} R_{\text{sp}}$. In order to take into account the power emitted by the device only, we define the apparent mirror loss $\tilde{\alpha}_m = (P_R + P_T + P_{\text{sp}}) / (v_g h \nu V_p N_p)$. This leads to the rate equation for optical amplifiers, involving the amount of power injected in amplifier P_I :

$$\frac{\partial N_p}{\partial t} = (\Gamma g - \alpha_i - \tilde{\alpha}_m) v_g N_p + \frac{P_I}{h \nu V_p} + \Gamma \beta_{\text{sp}} R_{\text{sp}}. \quad (2)$$

This expression is formally the same as the one generally used in the literature. However, using the definition of $\tilde{\alpha}_m$ and Eq. (2) at steady state ($\partial N_p / \partial t = 0$), we find that the apparent mirror losses must be expressed by

$$\tilde{\alpha}_m = (\Gamma g - \alpha_i) \frac{G_{\text{signal}} P_I + G_{\text{ASE}} G_{\text{sp}}}{(G_{\text{signal}} - 1) P_I + (G_{\text{ASE}} - 1) G_{\text{sp}}}, \quad (3)$$

and depend on the gain/absorption of the active region and on the power injected and generated in the device. This relation involves two gain factors: $G_{\text{signal}} = G_R + G_T$ is the total gain of the amplifier, with the gain in reflection (transmission) mode defined by $G_{R,T} = P_{R,T} / P_I$. The ratio between P_{sp} and G_{sp} is defined as the gain of the amplified spontaneous emission $G_{\text{ASE}} = P_{\text{sp}} / G_{\text{sp}} = P_{\text{sp}} / (h \nu V \beta_{\text{sp}} R_{\text{sp}})$. This factor indicates how much ASE power is extracted outside the cavity for the fraction of spontaneous emission that is radiated inside the cavity and in the optical mode.

The photon density in the active region can be calculated using Eq. (2), which allows us to calculate the stimulated recombination rate $R_{\text{st}} = v_g g N_p$. We find

$$R_{\text{st}} = \frac{1}{h \nu V} \frac{\Gamma g}{\Gamma g - \alpha_i} [(G_{\text{signal}} - 1) P_I + (G_{\text{ASE}} - 1) G_{\text{sp}}]. \quad (4)$$

Note that R_{st} is the sum of two factors that are proportional to the product between the inverse of the amplifier quantum efficiency¹⁰ $\Gamma g / (\Gamma g - \alpha_i)$, net power gain, and source term. This result makes obvious sense: for the input signal, the number of photons generated in the amplifier is proportional to the total emitted power $G_{\text{signal}} P_I$ minus the number of injected photons proportional to P_I .

To completely describe the properties of vertical-cavity devices, a rate equation for the carriers is needed and is generally given by $dN/dt = G_{\text{gen}} - R_{\text{rec}}$, where G_{gen} is the rate of injected electrons and R_{rec} is the rate of recombining electrons per unit volume in the active region.⁵ The rate equations for the carriers and the photons can be used to describe any vertical-cavity device provided the material

gain g can be considered as constant in the active region, which is a reasonable approximation in practice.

For the sake of completeness, let us give the analytical expressions of G_{signal} and G_{ASE} for a general VCSEA structure consisting of two DBRs surrounding a low-order cavity of total thickness L_c , in which is placed a thin active region. The DBRs are made of alternating high (n_2) and low (n_1) index layers of thickness L_2 and L_1 , respectively, with $\lambda_{\text{Bragg}} = 4n_1 L_1 = 4n_2 L_2$. The complex reflection coefficients of the front and back mirrors are defined as $r_{1,2} = \rho_{1,2} \exp(i\psi_{1,2})$, where $R_{1,2} = |r_{1,2}|^2 = \rho_{1,2}^2$ and $\psi_{1,2}$ is the phase of the mirror, which can be related to an equivalent penetration depth $L_{\text{pen}}^{1,2}$ of the electric field in the mirrors.⁵ An equivalent cavity length can then be defined as $L_{\text{eff}} = L_{\text{pen}}^1 + L_{\text{pen}}^2 + L_c$. The gain per pass is expressed as $G_s = \exp[(\Gamma g - \alpha_i) L_{\text{eff}}]$.

With these definitions, the total signal gain given by $G_{\text{signal}} = G_R + G_T$ can be easily calculated according to

$$G_{\text{signal}} = 1 + \frac{(1 - R_1)(1 + R_2 G_s)(G_s - 1)}{(1 - \sqrt{R_1 R_2} G_s)^2 + 4 \sqrt{R_1 R_2} G_s \sin^2 \Phi_0}, \quad (5)$$

where $\Phi_0 = 2\pi n_{\text{eff}} L_{\text{eff}} (\lambda_0^{-1} - \lambda_{\text{Bragg}}^{-1})$ is the single-pass phase detuning, n_{eff} the effective refractive index of the cavity, and λ_0 the wavelength of the signal to be amplified.

Calculation of gain G_{ASE} is a bit more complicated and will be presented elsewhere. We find:

$$G_{\text{ASE}} = \frac{\sqrt{G_s} \sum_{(i,j)=(1,2)} (1 - R_i)(1 + \sqrt{R_j G_s})^2}{2(1 + \sqrt{R_1 G_s})(1 + \sqrt{R_2 G_s})(1 - \sqrt{R_1 R_2} G_s)}. \quad (6)$$

At the lasing wavelength (determined by the condition $\Phi_0 = 2\pi$), gains G_{signal} and G_{ASE} become infinite when $G_s^{\text{th}} = 1/\sqrt{R_1 R_2}$. The apparent mirror loss given by relation (3) converges then to the usual factor $\Gamma g_{\text{th}} - \alpha_i = \alpha_m^{\text{th}} = (1/L_{\text{eff}}) \ln[1/\sqrt{R_1 R_2}]$, which was generally used for the modeling of resonant-cavity semiconductor amplifiers.^{1,3,8,11} Figure 1 displays the parameter $\alpha_m^{\text{th}}/\tilde{\alpha}_m$ ($G_s = 1, G_{\text{sp}} = 0$) and $\alpha_m^{\text{th}}/\tilde{\alpha}_m$ ($G_s = 1, P_I = 0$) versus the front and back mirror reflectivities as solid and dotted contour lines, respectively. These factors represent the error that is made using α_m^{th} instead of $\tilde{\alpha}_m$ in rate Eq. (2) at transparency ($G_s = 1$). For no input signal ($P_I = 0$) and for highly reflective mirrors, the apparent mirror losses $\tilde{\alpha}_m$ are very close to α_m^{th} , which can then be safely used in the rate equations for calculation of the amplified spontaneous emission. However, a study of expression (3) above transparency and below threshold in the case where the input signal is higher than the spontaneous emission ($P_I > G_{\text{sp}}$) shows that using α_m^{th} instead of $\tilde{\alpha}_m$ in Eq. (2) leads to an overestimate of the mirror losses by a factor that can be as high as 4. This result explains why the photon density calculated with the conventional rate equation^{1,3,8,11} leads to a systematic underestimate of the photon density compared to other methods based on Fabry-Pérot or traveling wave approaches, for example.^{4,6,7,12} In practice, this error leads to overestimating the saturation power of VCSEAs by, typically, 10 dB. The results presented in this letter are very general and can be applied to any Fabry-Pérot laser amplifier for which the material gain/absorption can be considered as constant in the structure. This key condition is usually satisfied for vertical-cavity devices. Note that the ma-

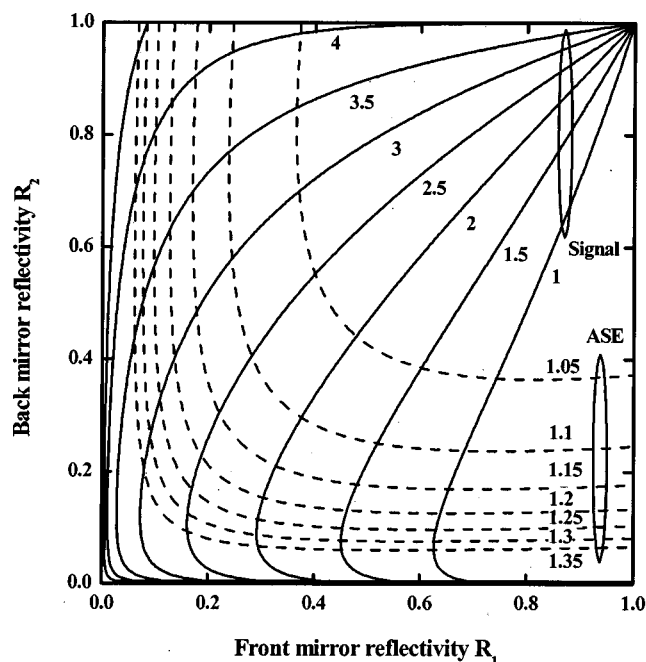


FIG. 1. Parameters $\alpha_m^{\text{th}}/\alpha_m$ ($G_s=1, G_{\text{sp}}=0$) and $\alpha_m^{\text{th}}/\alpha_m$ ($G_s=1, P_I=0$) plotted as solid and dotted contour lines, respectively, vs the front and back mirror reflectivities.

terial gain of in-plane Fabry-Pérot laser amplifiers cannot be considered as constant over the active region. Local expressions of the rate equations must then be used, as was proposed by Marcuse⁴ several years ago.

It is interesting to observe that the expression of the stimulated recombination rate due to the signal in Eq. (4) corresponds exactly to the result of Adams⁶ who calculated the averaged photon density in in-plane Fabry-Pérot laser amplifiers assuming a constant material gain throughout the cavity and using another approach based on traveling wave equations for the fields. Our expression for the stimulated recombination rate due to the ASE is, however, different: because of the thin active regions, the VCISOAs must be treated considering the fields rather than the intensities.

Note, finally, that using the rate equations given above, it is possible to simply calculate the quantum efficiency η of a RCEPD, which can be considered as a VCISOA with negative gain. The quantum efficiency of these devices is defined as the ratio between the power absorbed by the active region, and the power injected in the device, thus $\eta = -G_{\text{tot}}^-/P_I$.

Using the definitions presented above, and assuming an absorption per pass $G_s = \exp[-\Gamma g L_{\text{eff}}]$, we find that $\eta = 1 - G$. Using the relation $G = G_R + G_T$ with expression (5), we easily retrieve the result of Ünlü and Strite [see Eq. (8) of Ref. 13], with the only difference that the standing wave effect is automatically taken into account through our confinement factor Γ , which includes the gain/absorption enhancement factor.

In conclusion, the rate equation for photons has been revisited for vertical-cavity devices below threshold. We have demonstrated that the mirror loss expression, which is traditionally used in rate equations for photons in resonant-cavity semiconductor optical amplifiers, is incorrect below threshold because it does not satisfy the energy conservation. This error leads then to an underestimate of the photon density in the cavity, hence, to an underestimate of the stimulated recombination rate, and then to an overestimate of gain saturation effects for these amplifiers. The correct expression for the mirror losses has been derived and includes the amplified spontaneous emission. Finally, we have presented a unified way of describing vertical-cavity devices by photon rate equations, which can be applied to any device with stimulated gain or absorption like VCISOAs or RCEPDs, the only condition being that the material properties of the gain/absorbing medium are constant throughout the active region, which is generally reasonable for this kind of device.

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